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Natural Mass Hierarchy of Z Boson and Scalar Top in No-Scale Supergravity

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Abstract

It is studied that 'no-scale' model makes hierarchy between scalar top mass and Z boson mass naturally. The supersymmetry breaking parameters are constrained by flavor changing neutral currents in minimal supersymmetric standard model. One of the solution of the problem is that gaugino mass is the only source of the supersymmetry breaking parameters at Planck scale. However, in such scenario, we need a cancellation between Higgs mass parameters in minimization condition of the Higgs potential. We insist that there is no such cancellation in no-scale model, and that the no-scale model gives us the prediction of scalar top mass and lightest Higgs mass. The lightest Higgs mass is predicted as $m_H = 110 \pm 5$ GeV.

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1. Introduction

The supersymmetric theories now stand as the most promising candidate for the unified theory beyond the standard model [1]. The accurate data favor remarkably the supersymmetric grand unified theory (GUT) over the non supersymmetric theory [2]. The supersymmetry helps to resolve the gauge hierarchy problem [3]. In non supersymmetric standard models, squared Higgs mass receives quadratic divergent correction radiatively, and then we cannot explain the hierarchy between weak scale and the grand unified scale naturally. Supersymmetry removes the quadratic divergences and gives us a framework for naturally explaining the widely separated hierarchy.

In those contexts, the idea of a radiative breaking of the electroweak symmetry [4] is very popular. It is very attractive to explain the breaking of the electroweak symmetry through large logarithms between the Planck (or GUT) scale and the weak scale. The radiative corrections drive an up-type Higgs mass squared parameter negative for a large top Yukawa coupling, and thus the electroweak symmetry breaks down. The radiative symmetry breaking mechanism has consequences for the supersymmetric particle spectrum and gives us important constraints on the particle spectrum.

The constraints gives us a slight puzzle. In the radiative breaking mechanism, Z boson mass is related to supersymmetry breaking parameters. Thus, we believe that the supersymmetric particles does not so heavy compared with Z boson. However the experimental lower bounds for the supersymmetric particle masses are getting larger and larger day by day, and it seems that we require a fine tuning between the parameters in the Higgs potential [5].

It is well known that flavor changing neutral currents (FCNC) make important constraints to supersymmetry breaking scalar masses [1, 6]. We require that the scalar quark eigenmasses have degeneracy* to a few percent when the scalar masses are of the order of O(100) GeV. One of the solution of the scalar quark mass degeneracy is to consider the type of minimal gaugino mediation [7]. Namely, the gaugino mass is the almost only source for the supersymmetry breaking at Planck scale. The supersymmetry breaking parameters which have flavor indices are enough

*In addition to the degeneracy scenario, there is an alignment scenario, in which the scalar quark eigenvectors should be strongly aligned with those of the quark eigenvectors.

small compared to the gaugino mass at Planck scale, and become enough large due to renormalization group flow at low energy scale. Though this scenario is very attractive, such large gaugino masses cause the fine tuning described above to the Higgs potential. Are there any mechanism in which the fine tuning are not required even if the gaugino mass is large?

In this paper, we insist that the 'no-scale' supergravity model [12] does not require any fine tuning in the Higgs potential. The no-scale models are very suitable for the scenario of the minimal gaugino mediation. We will study the supersymmetric particle spectrum in no-scale models. In the no-scale models, the magnitude of the supersymmetry breaking parameters is also determined radiatively. We can investigate the theoretical upper bounds in the no-scale models, and we can judge the bounds at near future colliders. Especially, we insist that the no-scale models suggest us a mass hierarchy between Z boson mass and supersymmetry breaking masses naturally.

The organization of this paper is as follows. In Section 2, we review the unnatural tuning in the Higgs potential in Minimal Supersymmetric Standard Model (MSSM). In Section 3, we review no-scale supergravity models. In Section 4, we explain how we calculate the particle spectrum in our framework. In Section 5, we show the results for the particle spectrum and study its bounds. Finally, we conclude in Section 6 with a summary of our results.

2. Unnatural Tuning in Z Boson Mass

The tree level neutral Higgs potential in MSSM is given by

$$V^{(0)} = m_1^2|H_d^0|^2 + m_2^2|H_u^0|^2 - (m_3^2 H_d^0 H_u^0 + c.c.) + \frac{g^2+g'^2}{8}(|H_d^0|^2 - |H_u^0|^2)^2. \quad (2.1)$$

The Higgs mass parameters m_1^2 and m_2^2 are

$$m_1^2 = m_{H_d}^2 + \mu^2, \qquad m_2^2 = m_{H_u}^2 + \mu^2, \quad (2.2)$$

where $m_{H_d}^2$ and $m_{H_u}^2$ are soft supersymmetry breaking mass squared for the Higgs bosons, and μ is so-called Higgsino mass in the supersymmetric 'μ-term'. We denote the vacuum expectation values (VEVs) for H_d^0 and H_u^0 as v_d and v_u, respectively.

We require that electroweak symmetry breaks down, and then we find minimization conditions of the potential at tree level,

$$\frac{M_Z^2}{2} = -\mu^2 + \frac{m_{H_d}^2 - m_{H_u}^2\tan^2\beta}{\tan^2\beta - 1}, \quad (2.3)$$

$$\sin 2\beta = \frac{2m_3^2}{m_1^2 + m_2^2}, \tag{2.4}$$

where $\tan\beta = v_u/v_d$. We require that the Z boson mass M_Z is equal to 91 GeV and that $\tan\beta$ is not so close at 1 phenomenologically. We should mention here that the relation like Eq.(2.3) is usually satisfied even in the non-minimal models.

The radiative electroweak symmetry breaking occurs because $m_{H_u}^2$ is driven negative due to a large top Yukawa coupling in its renormalization group flow. It is well-known [8] that heavy gluino mass causes a weird cancellation among supersymmetry breaking Higgs mass squareds and μ in Z boson mass formula (2.3).

Let us make clear such an unnatural cancellation. The 'free' dimensionful parameters for MSSM is

$$\{m_0^2, M_{1/2}, A_0, B_0, \mu_0\}. \tag{2.5}$$

These parameters are introduced at Planck scale.* The main contribution for the negative $m_{H_u}^2$ is not original supersymmetry breaking scalar mass squared m_0^2 at GUT scale, but gluino mass $M_{\tilde{g}}$ [10, 5]. The scalar mass squared m_0^2 is insensitive to the negative $m_{H_u}^2$ in the regime $m_0 \sim O(100)$ GeV. Since the most sensitive parameters for the Higgs mass squared are the gaugino mass $M_{1/2}$ and μ among those parameters, the other three parameters are equal to zero for the time being. We show Fig.1† in which we plot the Z boson mass as a function of $\mu/M_{1/2}$ for given $M_{1/2}$. This figure provides us with the problem why the parameter μ is limited in the narrow range for an appropriate electroweak symmetry breaking, even when we solve the μ-problem‡. Besides, the parameter μ should be the right edge value in the figure for the allowed region, if the gaugino mass $M_{1/2}$ is larger than 200 GeV. In fact, the gaugino mass $M_{1/2}$ should be larger than about 200 GeV in the minimal gaugino mediation noted in Section 1.

Let us see the fine tuning in Eq.(2.3) in another point of view. Here we suppose that $\tan\beta$ is enough large ($\tan\beta > 3$) for only simplicity. Then the Z boson mass is written by

$$M_Z^2 = -2(\mu^2 + m_{H_u}^2) = -2m_2^2. \tag{2.6}$$

Since the parameters μ and $m_{H_u}^2$ are depend on scale Q, we should know the scale where we require the tuning between μ^2 and $m_{H_u}^2$. The scale is the one where 1-loop

*Since we would not like to consider the Physics beyond GUT, those parameters are given at GUT scale later.

†To plot the figure, we consider the 1-loop corrected scalar potential (3.2).

‡We have a problem which is so called μ-problem. The problem is why the supersymmetric parameter μ is the same order as supersymmetry breaking parameters.

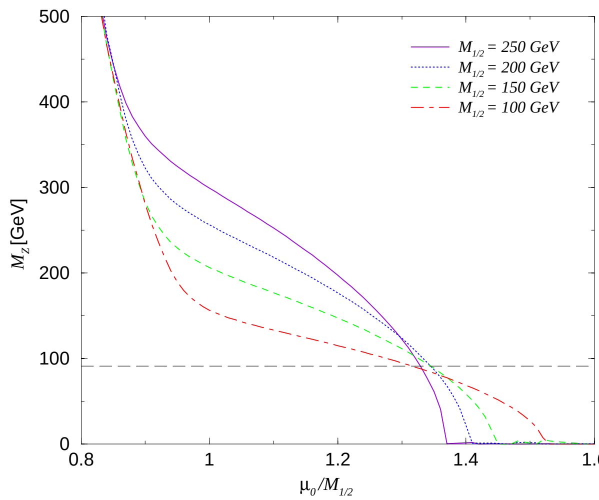

Figure 1: We show the Z boson mass as a function of $\mu_0/M_{1/2}$ for various gaugino masses. In this figure, we set m_0 and A_0 to be zero. We choose the B_0 parameter so as $\tan\beta = 10$ at the point $M_Z = 91$ GeV. In left side of this figure, $\tan\beta$ becomes close to 1, and the Higgs potential are destabilized.

corrected potential becomes small. We denote the scale as $Q_{\tilde{t}}$ since the scale is nearly to the mass of scalar top quarks. Then, the physical Z boson mass is approximated by

$$M_Z^2 \sim M_Z^2(Q_{\tilde{t}}), \tag{2.7}$$

where

$$M_Z^2(Q) \equiv -2m_2^2(Q). \tag{2.8}$$

We define the scale Q_0 where $M_Z^2(Q)$ vanish. The Q_0 is the scale where electroweak symmetry breaks down at tree level. Expanding $M_Z^2(Q)$ by $\ln Q$ around the scale Q_0, we obtain

$$M_Z^2 \sim 2\ln(\frac{Q_0}{Q_{\tilde{t}}})\frac{dm_2^2}{d\ln Q}(Q_0). \tag{2.9}$$

In this point of view, the fine tuning in Z boson mass is translated into the tuning between the scalar quark mass scale $Q_{\tilde{t}}$ and the scale Q_0. As stated in Ref.[9], electroweak symmetry breaks down only when the scale $Q_{\tilde{t}}$ is less than Q_0. This fact is the same thing that the parameter μ should be right edge in Fig.1 when the gaugino mass becomes greater.

There are many implication about naturalness in the literature. In Ref.[10], they point out that heavy scalar mass m_0 (which masses are of the order of 1 TeV) relaxes the fine tuning. In Ref.[8], they suggest that less fine-tuned model should be select as a candidate of scenario for supersymmetry breaking. It is preferred that the gaugino mass is not unified at GUT scale (e.g. D-brane model) in the reference. In those literature, we feel that the fine tuning in the Higgs potential is not dispelled. Are there any models which the cancellation occurs naturally?

In this paper, we will suggest that we have already had a model which explain the heavy gluino mass naturally without any fine tuning. The model is no-scale supergravity. In the folklore, it is said that more severe fine tuning is required in the no-scale supergravity model rather than ordinary models. We believe, however, that the interpretation is not correct. To see this, we will give a brief review of no-scale supergravity in the next section.

3. No-Scale Supergravity

In this section, we give a brief review of no-scale supergravity, and we see the natural mass hierarchy between Z boson and scalar top in the no-scale model.

In hidden sector model [11], we separate fields into two sectors, which are a visible sector and a hidden sector. The observable fields (quarks, leptons and Higgs fields) are involved in the visible sector. The hidden fields which break supersymmetry are living in hidden sector, and the hidden fields couple with observable fields through only gravitational interaction. The F terms of the hidden fields have VEVs due to the dynamics in only hidden sector in ordinary hidden sector models. In other words, the scale of supersymmetry breaking is determined with no relation to our visible sector. However, it is possible that a scalar potential for the hidden sector fields is flat at tree level, and VEVs of the hidden fields determine radiatively accompanied with visible sector dynamics. Such theories are called no-scale supergravity [12].

Let us see how gravitino mass is determined in no-scale model. Using the minimization conditions (2.3) and (2.4), we obtain the tree level MSSM scalar potential at the minimal point

$$V_{\rm mim}^{(0)} = -\frac{1}{2(g^2+g'^2)} M_Z^4 \cos^2 2\beta, \tag{3.1}$$

Since the Z boson mass is proportional to gravitino mass*, the potential involving the hidden sector is unbounded from below. However, there exists a 1-loop corrected

*We assume that the dimensionful parameters are proportional to gravitino mass. See Appendix.

scalar potential, namely

$$V^{(1)} = \frac{1}{64\pi^2} \sum_J (-1)^{2J}(2J+1) m_J^4 (\ln \frac{m_J^2}{Q^2} - \frac{3}{2}), \tag{3.2}$$

in $\overline{\mathrm{DR}}$ scheme [13]. As a result, the scalar potential is stabilized if Str $M^4 > 0$ [14], and the gravitino mass is determined dynamically.

We emphasize here that the gravitino mass is not independent on the visible sector parameter, namely μ, in the no-scale models. The naturalness argument in the no-scale model differs from arguments in the ordinary ones due to such dependence.

To confirm the natural hierarchy between Z boson mass and supersymmetry breaking masses, we will overview the minimization with respect to gravitino mass [12]. Since the total scalar potential is not depend on renormalization point, we evaluate the potential at scale where 1-loop corrected potential vanishes,

$$V^{(1)}(v_u, v_d; Q) = 0. \tag{3.3}$$

The scale is approximately the mass scale of scalar top quarks $Q_{\tilde{t}}$,

$$Q_{\tilde{t}} \equiv (m_{\tilde{t}_1} m_{\tilde{t}_2})^{1/2}. \tag{3.4}$$

Then we find the minimal value of the effective scalar potential [12],

$$V_{\min} \sim -C Q_{\tilde{t}}^4 \left(\ln \frac{Q_{\tilde{t}}^2}{Q_0^2} \right)^2, \tag{3.5}$$

where Q_0 is the scale where electroweak symmetry breaks down at tree level, and C is a constant. In no-scale model, $Q_{\tilde{t}}$ is determined by which the $V_{\min}$ is minimized. Minimizing $V_{\min}$ for $Q_{\tilde{t}}$, we find that the scale $Q_{\tilde{t}}$ is determined as

$$\ln \frac{Q_0^2}{Q_{\tilde{t}}^2} = 1. \tag{3.6}$$

It is important that the scale $Q_{\tilde{t}}$ is very close to the scale Q_0,

$$Q_{\tilde{t}} = Q_0 / e^{1/2}. \tag{3.7}$$

Substituting it to Eq.(2.9), we find Z boson mass formula in no-scale model as follows in large $\tan\beta$,

$$M_Z^2 \sim \frac{d}{d \ln Q} m_2^2. \tag{3.8}$$

The 1-loop renormalization group equations (RGEs) for $m_{H_u}^2$ and μ^2 are following,

$$\frac{d}{d\ln Q}m_{H_u}^2 = \frac{1}{2\pi}[3\alpha_t(m_{\tilde{t}_3}^2 + m_{\tilde{t}}^2 + m_{H_u}^2 + A_t^2) - (\alpha' M_1^2 + 3\alpha_2 M_2^2)], \tag{3.9}$$

$$\frac{d}{d\ln Q}\mu^2 = \frac{1}{2\pi}[3\alpha_t + 3\alpha_b + \alpha_\tau - (\alpha' + 3\alpha_2)]\mu^2, \tag{3.10}$$

where $\alpha_t = Y_t^2/4\pi$ and Y_t is a top Yukawa coupling. It turns out that the Z boson mass is determined hierarchically compared to the supersymmetry breaking masses, and that the hierarchy is characterized by 1-loop factor $3\alpha_t/2\pi$. This fact is what we insist in this paper.

The Eq.(3.8) is easily extended in the case of general $\tan\beta$. Expanding Eq.(2.3) by $\ln Q$ around Q_0, we obtain the following Z boson mass formula at tree level

$$M_Z^2 \cos^2 2\beta \sim \dot{m}_1^2 \cos^2\beta + \dot{m}_2^2 \sin^2\beta - \dot{m}_3^2 \sin 2\beta, \tag{3.11}$$

where $\dot{m}_i^2 = dm_i^2/d\ln Q$. The relation will be tested in future experiments.

In introducing this formula, we neglect the derivative of 1-loop corrected potential with respect to Higgs VEVs in Z boson mass formula. Since it is complicate to write down the derivative, we will calculate 1-loop corrected relation numerically. We will show how we obtain our numerical results in next section.

4. Methods

We concentrate the following effective scalar potential with Higgs VEVs independent shift,

$$V_{\text{eff}}(v_u, v_d) = V^{(0)}(v_u, v_d; Q) + V^{(1)}(v_u, v_d; Q) - V^{(1)}(v_u, v_d = 0; Q). \tag{4.1}$$

This potential is independent of the renormalization point Q at 1-loop level schematically [15]. In the expression, $V^{(0)}$ is a tree level potential (2.1) and $V^{(1)}$ is a 1-loop correction (3.2) of the potential.

At first, we show the effective potential which is minimized by Higgs VEVs v_d and v_u (Fig.2). This figure is drawn in the minimal case where $m_0^2 = 0$ and $A_0 = B_0 = 0$. The horizontal axis is for gaugino mass. We can see that there is a minimum with respect to the gaugino mass.

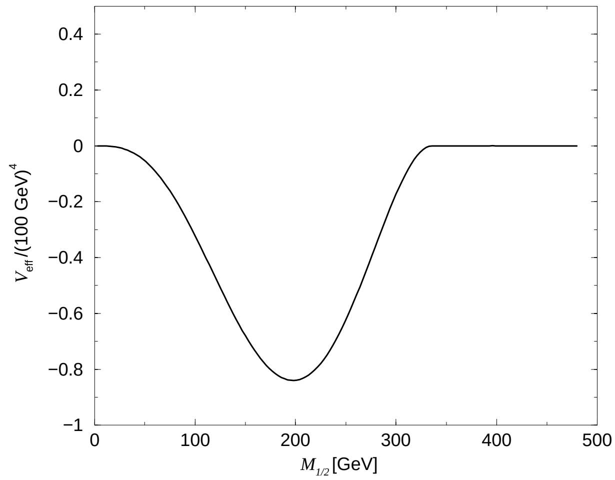

Figure 2: We show the effective potential Eq.(4.1) minimized by Higgs vacuum expectation values, v_u and v_d.

Since the gaugino mass is the most sensitive parameter for the supersymmetry breaking Higgs mass squared, we normalize the following dimensionful parameters of MSSM,

$$\{m_0^2, M_{1/2}, A_0, B_0, \mu_0\}, \tag{4.2}$$

divided by the gaugino mass $M_{1/2}$, and we adopt the following four dimensionless parameters

$$\{\hat{m}_0^2, \hat{A}_0, \hat{B}_0, \hat{\mu}_0\} \tag{4.3}$$

as parameters for no-scale models. The *hat* is denoted the the parameters are normalized by the gaugino mass (squared). The gaugino mass is determined in minimizing the potential if we fix the hatted parameters*. The freedom for $\hat{\mu}_0$ is consumed when the Z boson mass is fixed as 91 GeV. If we fix $\tan\beta$, the $\hat{B}_0$ is consumed and the remaining free parameters are only $\hat{m}_0$ and $\hat{A}_0$.

To show our numerical analysis, we evolve the supersymmetry breaking parameters with the full two-loop RGEs [16].

Since the potential does not depend on the renormalization point ideally, we may choose any scale. Nevertheless, we minimize the potential (4.1) near the scale where the electroweak symmetry breaks down *at tree level* to fix our aim. This

*The gravitino mass $m_{3/2}$ is also parameter of the model, but it is still a free parameter since the proportional coefficient $m_{3/2}/M_{1/2}$ is not determined in the model.

is because we must consider the threshold effect for the supersymmetric particles, for instance, scalar quarks and gluinos. Therefore, we adopt our method in the following. Firstly, we minimize the effective potential with respect to Higgs VEVs and gaugino mass at the scale above those supersymmetric particle masses. After the minimization, we include the one-loop threshold corrections from supersymmetric particles [17], and fix the physical quantities. We take as inputs $\alpha_{\rm em}^{-1}(M_Z) = 127.9$, $\sin^2\theta_W(M_Z)_{\overline{\rm MS}} = 0.2309$ and $M_Z = 91.2$ GeV.

The strong gauge coupling has a discrepancy between the prediction from GUT and experimental measurement. The value of the strong gauge coupling α_3 is predicted as $\alpha_3(M_Z) = 0.13$ in GUT, while in the experimental measurement $\alpha_3(M_Z) = 0.119$. We adopt the experimental value for the strong gauge coupling. The resulting particle spectra have much dependence upon the strong gauge coupling and top Yukawa coupling. For smaller gauge coupling, the supersymmetric particles become heavier. This is mainly because the top Yukawa coupling at GUT scale is bigger for smaller gauge coupling.

We assume that the gaugino masses are unified at GUT scale[†]. We also assume the universality of m_0^2 and A_0 for their flavor and matter indices at GUT scale for simplicity.

The bottom quark mass and tau lepton mass is fixed as $m_b(M_Z) = 3.0$ GeV and $m_\tau(M_Z) = 1.7$ GeV. The results have little dependence for the bottom and tau masses. The top quark pole mass is fixed as $M_t = 174$ GeV. The 1-loop relationship between the pole mass and tree level mass $Y_t v_u$ is given by $M_t = Y_t v_u(1 + 5\alpha_3/3\pi)$ in $\overline{\rm DR}$ scheme.

5. Numerical Results

It is convenient that we present RGE solution by the following parameterization [5]. The dimensionful parameters at low energy are written by using GUT scale parameters.

First of all, the up-type Higgs mass squared m_2^2 is written as

$$m_2^2 = 1.0\mu_0^2 - 0.05m_0^2 - 1.75M_{1/2}^2 - 0.34M_{1/2}A_0 - 0.10A_0^2, \tag{5.1}$$

in the case of $\tan\beta = 10$. The mass of Z boson is $M_Z^2 \sim -2m_2^2$. It is easy to see that we require the fine tuning between μ_0 and $M_{1/2}$, if the gaugino is much heavier

[†]The GUT scale is defined as the scale where $\alpha_1 = \alpha_2$, which are gauge coupling constants.

than Z boson. It is worth noting that the coefficient of m_0^2 is very small in the RGE solution in the expression of m_2^2. This is because the 'focus point scale' for $m_{H_u}^2$ is of the order of 100 GeV [10]*.

In contrast, the RGE solution for $dm_2^2/d\ln Q$ is

$$\frac{dm_2^2}{d\ln Q} = 0.015\mu_0^2 + 0.026m_0^2 + 0.245M_{1/2}^2 - 0.011M_{1/2}A_0 - 0.004A_0^2, \tag{5.2}$$

when we take $\tan\beta = 10$. The Z boson mass in no-scale model is $M_Z^2 \sim dm_2^2/d\ln Q$. We can easily see that the tuning required above is not necessary in the no-scale model.

These two Eqs.(5.1) (5.2) are important to understand our numerical results shown below qualitatively, such that $-2m_2^2 \sim dm_2^2/d\ln Q = (100 - 110\text{GeV})^2$. The numerical value (100-110 GeV) for M_Z is caused by 1-loop corrected potential [18].

We will show the numerical results of minimization of the potential including 1-loop corrected potential. In the following figures, the sign of the μ parameter is positive in the notation in Eq.(A.4).

In Fig.3, we show the contour plot for gaugino mass $M_{1/2}$ as a function of m_0 and A_0 in the case of $\tan\beta = 10$. Shaded area in the right side in the figure is excluded for the condition $M_{1/2} > 100$ GeV, which means that the lightest chargino is heavier than 85 GeV. Upper and lower areas are excluded for charge and color breaking (CCB), namely,

$$\begin{aligned} A_t^2 &> 3(m_{\tilde{q}_3}^2 + m_{\tilde{t}}^2 + m_{H_u}^2), \\ A_b^2 &> 3(m_{\tilde{q}_3}^2 + m_{\tilde{b}}^2 + m_{H_d}^2), \\ A_\tau^2 &> 3(m_{\tilde{\ell}_3}^2 + m_{\tilde{\tau}}^2 + m_{H_d}^2). \end{aligned} \tag{5.3}$$

In the black area at $m_0 \sim 0$ and $A_0 \sim -500$ GeV, the right-handed scalar tau is lighter than lightest neutralino, which is not preferred for neutralino LSP. We note that the m_0 and A_0 are tuned in the lower right region in the figure. We prefer the small m_0 because of FCNC constraints. Therefore, we do not regard the large m_0 region . In Fig.4, we show m_0-$M_{1/2}$ plot at $A_0 = 0$. We understand its elliptic shape from the Eq. (5.2) qualitatively.

In Fig.5, we show the chargino masses as a function of m_0 for various A_0. We remark that the dots are plotted every 0.2 intervals for $\hat{m}_0$ (not m_0), and 0.5 intervals

*In the reference [10], it seems that the coefficient of m_0^2 is opposite sign to ours. In our calculation, the sign is reversed when we take the top quark pole mass as $M_t = 172$ GeV.

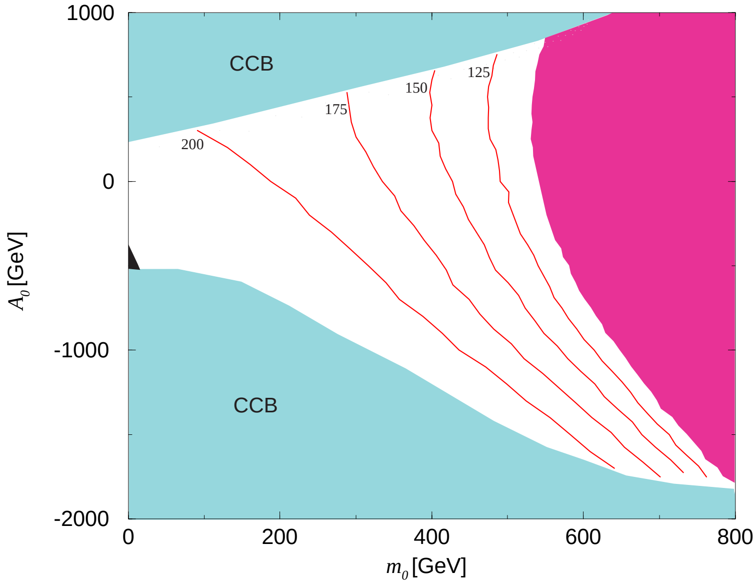

Figure 3: We show the contour plot for gaugino mass as a function of m_0 and A_0 in the case of $\tan\beta = 10$.

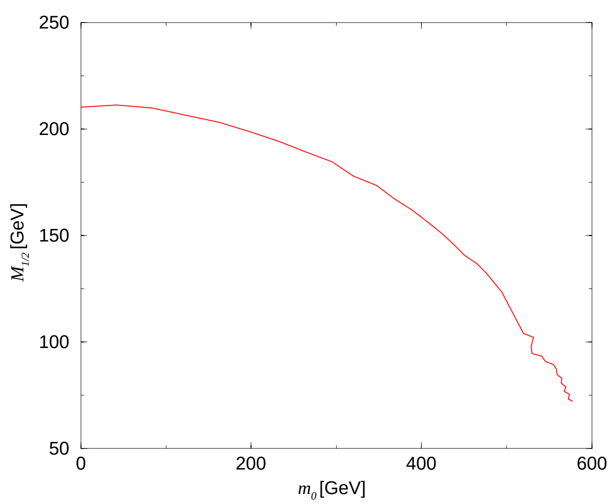

Figure 4: We show m_0-$M_{1/2}$ plot at $A_0 = 0$ in the case of $\tan\beta = 10$.

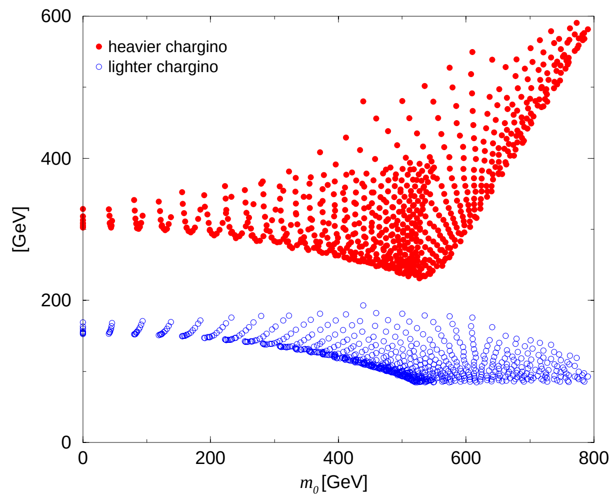

Figure 5: We show the chargino masses. The heavier and lighter chargino masses are approximately μ and wino mass M_2 respectively. We cut the lighter chargino mass which is smaller than 85 GeV.

for $\hat{A}_0$, thus the density of the dots is not related to probability of the parameters. This remark is also applied in the figure below. In Fig.6, we show the gluino mass, lightest chargino mass and lightest neutralino mass as a function of m_0 for various A_0 in the same way of Fig.5.

The important prediction for the no-scale model is that scalar top masses are almost determined independently of the scalar mass m_0. We plot the scalar top masses in Fig.7.

In supersymmetric models, lightest Higgs mass is bounded by M_Z at tree level. However this upper bound is corrected by 1-loop potential [19]. Since the scalar top masses are almost determined, the lightest Higgs mass is also predictable in no-scale model. We plot the lightest Higgs mass for $\tan\beta = 5$, 10, 30 in Fig.8. It is important that the lightest Higgs mass is 110 ± 5 GeV for small m_0. The small m_0 is favored for FCNC constraints. In calculating the lightest Higgs mass, we adopt the 2-loop approximate formula for the mass in the Ref.[20]. We line the recent LEPII bound on the non observation of $e^+e^- \to ZH$ [21] for one's information.

6. Discussion

In order to see our insist visibly, we show the figure (Fig.9) in the corresponding

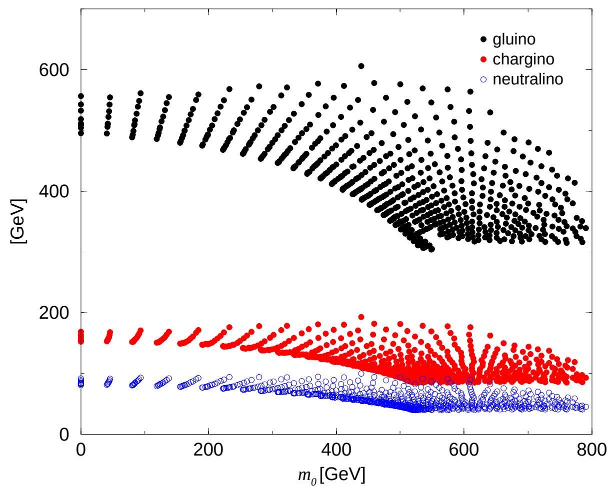

Figure 6: We show the gluino mass, lightest chargino mass and lightest neutralino mass. The 1-loop correction for gluino mass is included. We cut the lighter chargino mass which is smaller than 85 GeV.

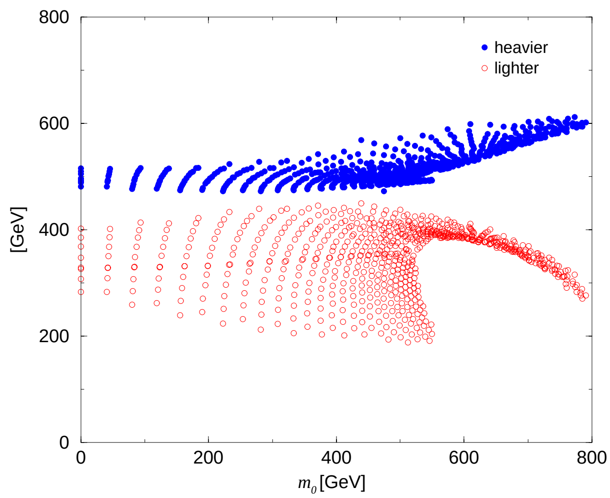

Figure 7: We show the scalar top masses. The scalar top masses are determined up to left-right mixing.

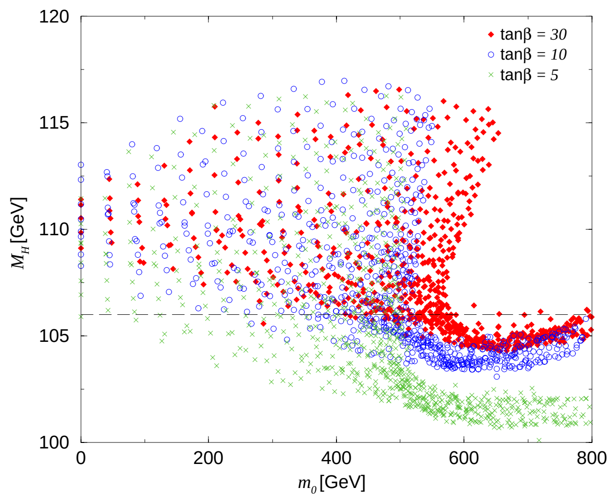

Figure 8: We plot the lightest Higgs mass for $\tan\beta = 5{,}10{,}30$. The dots are plotted every 0.2 intervals for $\hat{m}_0$ (not m_0), and 0.5 intervals for $\hat{A}_0$. The density of the dots are not related to probability of the parameters.

plot to Fig.1. Again, we set the parameter m_0^2 and A_0 to be zero. We choose the B_0 parameter so as to be $\tan\beta$ =10 at the point M_Z =91 GeV*. We plot the Z boson mass as a function of $\mu_0/M_{1/2}(=\hat{\mu}_0)$. There is not a weird constraint for the parameter $\hat{\mu}_0$ for electroweak symmetry breaking, contrary to the case in Fig.1. Therefore, the model building God can create the MSSM parameters without considering whether electroweak symmetry can break down at low energy. Our Z boson mass (91 GeV) does not lie on a special point, contrary to the ordinary case.

The following quantity [22] is usually used for measuring the sensitivity of the Z boson mass to variations in a parameter a,

$$\Delta_a = \left| \frac{\partial \ln M_Z^2}{\partial \ln a} \right|. \tag{6.1}$$

The value of $\Delta_{\hat{\mu}_0}$ is large, namely the Z boson mass is sensitive to the parameter $\hat{\mu}_0$ in also no-scale model. However, the large value of $\Delta_{\hat{\mu}_0}$ in no-scale model does not cause any fine tuning problem, contrary to ordinary models. It is just an event that a value of Z boson mass is selected.

Some people may say that it is just an event also in other supersymmetry breaking scenarios. The opinion is obviously true. However, the predictive ability in

*Incidentally, $\tan\beta$ is just 10 when the B_0 parameter is equal to zero.

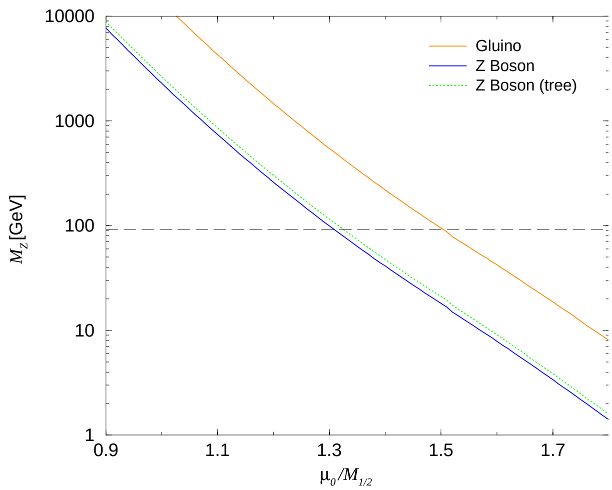

Figure 9: We show the Z boson mass and gluino mass as a function of $\mu_0/M_{1/2}$ in the case of no-scale model. Their mass ratio is approximately constant to $\mu_0/M_{1/2}$. We also plot the tree level Z boson mass formula Eq.(3.8).

no-scale model is completely different. The subtractive tuning in ordinary model does not have any predictive power. The supersymmetry breaking mass scale may be of the order of 10 TeV in the ordinary model. On the other hand, we do not require any subtractive tuning in the no-scale model, and we predict that all the supersymmetric particles (except gravitino) appear below about 500-600 GeV. Especially, we can judge the no-scale model when we search the Higgs boson or gauginos near future. This predictive ability is our motivation of the no-scale model. For theoretical physicists, it is important to search predictive models. To say more, it is important that we recognize that the fine-tuning in the Higgs potential may impose the no-scale supergravity, and we investigate the prediction of the no-scale models. This is a process of Physics to access unknown world.

Note added: While completing this paper, we received a paper by R. Barbieri and A. Strumia [23] which also considers that the electroweak breaking scale gets related to supersymmetry breaking scale by a loop factor in a similar way to us.

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A Notation and Convention

The superpotential of minimal supersymmetric standard model (MSSM) is presented as

$$W = Y_u Q \cdot H_u U^c + Y_d H_d \cdot Q D^c + Y_e H_d \cdot L E^c + \mu H_d \cdot H_u, \tag{A.1}$$

where the SU(2) inner product is defined as

$$H_d \cdot H_u \equiv H_d^T \epsilon H_u, \qquad \epsilon = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}. \tag{A.2}$$

Here, Q, U^c, D^c, L, E^c are matter chiral superfields, and H_u and H_d are Higgs doublets.

We denote soft supersymmetry breaking terms as

$$\begin{aligned} V_{soft} &= m_{H_d}^2 |H_d|^2 + m_{H_u}^2 |H_u|^2 \\ &+ m_{\tilde{q}}^2 \tilde{q}\tilde{q}^\dagger + m_{\tilde{u}}^2 \tilde{u}_R \tilde{u}_R^\dagger + m_{\tilde{d}}^2 \tilde{d}_R \tilde{d}_R^\dagger + m_{\tilde{\ell}}^2 \tilde{\ell}\tilde{\ell}^\dagger + m_{\tilde{e}}^2 \tilde{e}_R \tilde{e}_R^\dagger \\ &+ (A_u Y_u \tilde{q} \cdot H_u \tilde{u}_R^c + A_d Y_d H_d \cdot \tilde{q}\tilde{d}_R^c + A_e Y_e H_d \cdot \tilde{\ell}\tilde{e}_R^c + h.c.) \\ &+ (B\mu H_d \cdot H_u + h.c.) \end{aligned} \tag{A.3}$$

To make clear our notation, we present left-right component in the scalar top quark mass matrix and chargino mass matrix in the following. The left-right mixing is

$$(A_t + \mu \cot\beta) m_t. \tag{A.4}$$

The chargino mass matrix is presented as

$$M_{\chi^+} = \begin{pmatrix} M_2 & \sqrt{2} M_W \cos\beta \\ \sqrt{2} M_W \sin\beta & -\mu \end{pmatrix}. \tag{A.5}$$

The supergravity theories are given by Kähler potential K, superpotential W and gauge kinetic function f. The scalar potential is given in supergravity as

$$V = e^K [g^{ij^*} (D_i W)(D_{j^*} W^*) - 3 W W^*]. \tag{A.6}$$

Using Kähler transformation $G = K + \log W + \log W^*$, we obtain

$$V = e^G [G^i G_i - 3]. \tag{A.7}$$

No-scale Kähler potential [12] is written as

$$G = -3 \ln(T + \bar{T} - h(\phi_i^*, \phi_i)) + \ln |W(\phi_i)|^2, \tag{A.8}$$

where T is a moduli field and ϕ are fields in the visible sector. The function h is a Kähler potential for the visible fields. Then the scalar potential is

$$V = \frac{3|W|^2}{(T+\bar{T}-h(\phi_i^*, \phi_i))^2}\left|\frac{\partial W}{\partial \phi_i}\right|^2 . \tag{A.9}$$

If the global supersymmetric conditions $\partial W/\partial\phi_i = 0$ are satisfied, the scalar potential for T is flat and the gravitino mass $m_{3/2} = e^{G/2}$ is not determined.

Expanding Kähler potential with respect to visible fields Q, we write the Kähler potential [24] in general

$$K = \hat{K}(T, T^*) + \tilde{K}_{ij^*}(T, T^*)Q^iQ^{j^*} + \frac{1}{2}(H_{ij}(T, T^*)Q^iQ^j + h.c.) + \cdots, \tag{A.10}$$

where T^α's are hidden sector fields (Dilaton and Moduli). The superpotential is given by

$$W = \hat{W} + \tilde{\mu}_{ij}Q^iQ^j + \tilde{Y}_{ijk}Q^iQ^jQ^k + \cdots. \tag{A.11}$$

Substituting VEV into T, we obtain the effective superpotential in flat limit

$$W_{\text{eff}}(Q) = \mu_{ij}Q^iQ^j + Y_{ijk}Q^iQ^jQ^k. \tag{A.12}$$

The parameters μ and Y are written as [24]

$$\mu_{ij} = e^{\hat{K}/2}\frac{\hat{W}^*}{|\hat{W}|}\tilde{\mu}_{ij} + m_{3/2}(H_{ij} - G^{\alpha^*}\partial_{\alpha^*}H_{ij}), \tag{A.13}$$

$$Y_{ijk} = e^{\hat{K}/2}\frac{\hat{W}^*}{|\hat{W}|}\tilde{Y}_{ijk}. \tag{A.14}$$

In order to solve the μ-problem, we often set the $\tilde{\mu}$ to be zero [25]. Then the μ parameter in the flat limit is proportional to the gravitino mass.

Gaugino mass is given by gauge kinetic function f_a as

$$M_a = \frac{1}{2}m_{3/2}G^\alpha\partial_\alpha f_a. \tag{A.15}$$

Supersymmetry breaking scalar mass squared

$$m^2_{ij^*} = m^2_{3/2}(\tilde{K}_{ij^*} - G^\alpha G^{\beta^*}R_{\alpha\beta^*ij^*}). \tag{A.16}$$

Those two parameters are proportional to gravitino mass (squared).

The A and B parameters are complicated and they are not necessarily proportional to gravitino mass. In this paper, we suppose the A and B are proportional to the gravitino mass for simplicity.

We note that the parameters m_0, A and B are zero at Planck scale in strict no-scale model which is given by the Kähler potential (A.8). In this paper, we loosen the boundary condition.

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